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Quantum State Tomography of Single Qubit Using Density Matrix

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Abstract. The quantum state tomography is a fundamental part in the development of quantum technologies. It can be used to know the signal characterization of small particle called photon in the nanoscale. In this study, photon number has been measured in order to produce the states tomography. Optical devices and quantum-mechanical approaches were explored to obtain the quantum state tomography. Due to a single qubit state density matrix can be revealed by Stokes parameters, so there are four set-ups to measure the Stokes parameters for each sample. The density matrix is used because the pure state only appear theoreticaly. In the real experiment, It always exibits a mixed state. The samples of tomography measurements consist of linear state, cicular state and the IR 808nm. In this study, state tomography is shown by 2x2 density matrix. This experiment also provides the fidelities of experiment result. And it shows the good agreement. From this experiment, the state of IR 808nm has been detected. The laser that examined is showing a vertical state with fidelity F=97,34%.
Keywords: a single qubit, density matrix, quantum state

INTRODUCTION

Quantum physics is capable to reveal the behavior of particles in nanoscale that classical physics cannot describe (Gillespie, 1988). A Research in the nanophotonics field has positive contribution for future technology. In this digital era, technology is getting smaller and smaller in size, but it has more efficient performance. Lack of nanotechnology manufacturing and analysis tools are the roadblock to innovation, as is lack of modeling tools. New characterization methodes are required to study nanoscale interaction. (Pomrenke, 2004).

Quantum computing is the use of a superposition and entanglement. In analogy to the primary unit of information in computer science, the state of a quantum system is labeled a "quantum bit" or "qubit". (Niggebaum, 2011). This concept, a qubit, is a component of quantum mechanics. A pure qubit state is a coherent superposition of the basis states. This means that a single qubit can be expressed by a linear combination of $|0\rangle$ and $|1\rangle$. Improvement in realization of the properties of state, especially about state tomography, the quantum computer can be developed. The tomography of a quantum state is a process of creating an image of a signal. The laser which is fired certainly has a quantum state and the image can be produced. The observer can not see the quantum state of the laser directly and also can not measure it by using an instan tool. So, the optical set-up will be needed.

MATERIALS AND METHODS

Polarization

Light, such as any other electromagnetic wave, roughly always propagates as a transverse wave, with both electric and magnetic fields oscillating perpendicularly to the direction of propagation. The direction of the electric field is called the polarization of the wave (Monteiro, 2016). A wave polarized linearly by a wave plate has the same electric field direction. The polarization states can be represented in a two dimensional vector space (Rothberg, 2008). Polarized light in the *x* (horizontal) direction can be represented by $|H\rangle$ and in the *y* (vertical) direction by $|V\rangle$. $|H\rangle$ is acting for horizontal state and $|V\rangle$ is acting for vertical state. These vectors, $|H\rangle$ and $|V\rangle$ are called *kets*, a name proposed by Paul Dirac. It is part of the word *bracket*. In quantum mechanics, it is called *Dirac Notation*. The polarization state in general is:

$$|\psi\rangle = a|H\rangle + b|V\rangle$$

where *a* and *b* are in general complex with

$$|a|^2 + |b|^2 = 1$$

Then $|\psi\rangle$ is normalized. In the case of linear polarization *a* and *b* are real numbers. Those numbers can be written as $\cos\theta$ and $\sin\theta$. $|H\rangle$ and $|V\rangle$ are the basis states. And those two basis are an orthonormal basis because $|H\rangle$ and $|V\rangle$ are perpendicular. Another orthonormal basis can be described as follow:

$$|\theta\rangle = \cos\theta|H\rangle + \sin\theta|V\rangle$$

$$|\varphi\rangle = -\sin\varphi|H\rangle + \cos\varphi|V\rangle$$

With $\theta \perp \varphi$ (perpendicular). In other case, the type of polarized light can be circular polarization. There are two types in circular polarization. The first is right circular polarization and the second is left circular polarization. Right circular polarization can be dicribed as:

$$|R\rangle = \frac{1}{\sqrt{2}}|H\rangle + i|V\rangle$$

And left circular polarization can be described as follow:

$$|L\rangle = \frac{1}{\sqrt{2}}|H\rangle - i|V\rangle$$

The circular polarizations have complex coefficients in this case. And that is the difference between linear and circular polarization. To represent the state vectors, it needs matrices representation. In this case, Jone's calculus can represent the polarization states (Fowles, 1975) as follows:

$$\left|H\right\rangle = \begin{pmatrix}1\\0\end{pmatrix} ; \left|V\right\rangle = \begin{pmatrix}0\\1\end{pmatrix} ; \left|D\right\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\1\end{pmatrix} ;$$

$$\left|A\right\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\-1\end{pmatrix} ; \left|R\right\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\-i\end{pmatrix};$$

$$\left|L\right\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\i\end{pmatrix}$$

The density matrix or density operator can be used to represent the state of a quantum system. The density matrix is used because it is a practical tool when dealing with mixed states. A pure state is a state characterized by a single wave function. Whereas mixed states are a mixture of statistics that have imperfect information or mix information about systems, which are used to obtain quantum states. The density matrix of pure state is formally defined as the outer product of the wavefunction and its conjugate (Henao, 2017):

$$\rho_H = \left|H\right\rangle\left\langle H\right| = \begin{pmatrix}1 & 0\\0 & 0\end{pmatrix}$$

$$\rho_V = \left|V\right\rangle\left\langle V\right| = \begin{pmatrix}0 & 0\\0 & 1\end{pmatrix}$$

$$\rho_D = \left|D\right\rangle\left\langle D\right| = \begin{pmatrix}1 & 1\\1 & 1\end{pmatrix}$$

$$\rho_A = \left|A\right\rangle\left\langle A\right| = \begin{pmatrix}1 & -1\\-1 & 1\end{pmatrix}$$

$$\rho_R = \left|R\right\rangle\left\langle R\right| = \frac{1}{2}\begin{pmatrix}1 & i\\-i & 1\end{pmatrix}$$

$$\rho_L = \left|L\right\rangle\left\langle L\right| = \frac{1}{2}\begin{pmatrix}1 & -i\\i & 1\end{pmatrix}$$

A single qubit state density matrix can be related by four parameters called Stokes Parameters (James, On the Measurement of Qubit, 2008), As follow,

$$\rho = \frac{1}{2S_0}\sum_{i=0}^{3} S_i\sigma_i$$

σ_i are the standart Pauli matrices (σ_1, σ_2, σ_3) plus the identity (σ_0). And S_i are the Stokes parameters (S_0, S_1, S_2, S_3).

We can represent the polarization states by state vectors in a two-dimensional vector space (Rothberg, 2008). In this case, an optical setup consisting of waveplates and polarizers are used, to perform projective measurements to different prepared polarization states. The half-wave plate is an optical device which changes the direction into linear polarization.

The effect of the half-wave plate (HWP) and quarter-wave plate (QWP) can be writen the matrix as follow (James, On the Measurement of Qubit, 2008):

$$\hat{O}_{HWP} = \begin{pmatrix}\cos 2\theta & -\sin\theta\\-\sin 2\theta & -\cos 2\theta\end{pmatrix}$$

$$\hat{O}_{QWP} = \frac{1}{\sqrt{2}}\begin{pmatrix}i-\cos 2\varphi & \sin 2\varphi\\\sin 2\varphi & i+\cos 2\varphi\end{pmatrix}$$

The QWP converts linearly polarized light into circularly polarized light. To make sure that the calculation is compatible with the optical equipment, simulating by experiment is needed to prove the functions. Probabilities calculation can be completed using mathematical preparations. For example, the representation of polarization phenomenon is shown that the light passes through an optical device as operator. It can be described as bra-ket notation with normalized state (Bjork, 2003) as follows:

$$\left\langle H\right|\hat{O}_{\rho H}\left|H\right\rangle = 1$$

If the system is orthogonal, the equation will equal zero, as follows:

$$\left\langle V\right|\hat{O}_{\rho H}\left|H\right\rangle = 0$$

$\hat{O}_{\rho h}$ is an operator which represents the optical device like polarizer beam splitter gives two output as matrix because polarizer beam splitter can reflect and transmit the light. the transmitted light will be a horizontal state and the reflected light will be a vertical state (Rizea, 2011). So, the matrices as follow:

$$\hat{O}_{\rho H} = \begin{pmatrix}1 & 0\\0 & 0\end{pmatrix} ; \hat{O}_{\rho V} = \begin{pmatrix}0 & 0\\0 & 1\end{pmatrix}$$

And $|H\rangle$ and $|V\rangle$ is a ket vector of polarizing beamsplitter which tell us the polarization type.

In other case, when the polarized light $|H\rangle$ passes throught the quarter waveplate and polarizing beamsplitter, we can write as normalized state sa follows:

$$\left\langle H\right|\hat{O}^{\dagger}\hat{O}_{\rho H}\hat{O}\left|H\right\rangle$$

However, all measurable or observable entities correspond to observables, $\hat{O}$, which are Hermitian operators. (Bjork, 2003) The definition of Hermitian operator is:

$$\hat{O}^{\dagger} = \hat{O}$$

The probabilities from that calculation are square of the result. The relation between the probability and rotaion angle of wave plate will shown in this graph:

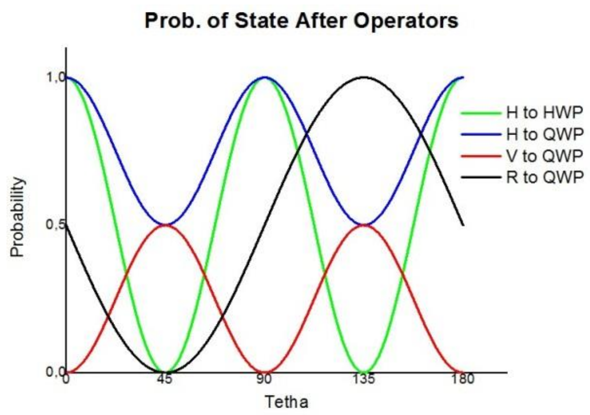

Figure 1. The graph of relation between probability and angle of waveplat (base on calculation)

The graph shows that probabilities will be changed when the angle of wave plate is rotated. The first curve can be writen with the formulations as follow:

$$\langle H|\hat{O}^{\dagger}_{HWP}\hat{O}_{\rho H}\hat{O}_{HWP}|H\rangle$$

If the equations are subtituted to the form above, it will be:

$$\begin{pmatrix}1 & 0\end{pmatrix}\begin{pmatrix}\cos 2\theta & -\sin 2\theta \\ -\sin 2\theta & -\cos 2\theta\end{pmatrix}\begin{pmatrix}1 & 0 \\ 0 & 0\end{pmatrix}\begin{pmatrix}\cos 2\theta & -\sin 2\theta \\ -\sin 2\theta & -\cos 2\theta\end{pmatrix}\begin{pmatrix}1 \\ 0\end{pmatrix}$$

The fuction of that calculation is:

$$\cos^2 2\theta$$

This fuction represent the curve graph (H to HWP). And the second curve is:

$$\langle H|\hat{O}^{\dagger}_{QWP}\hat{O}_{\rho H}\hat{O}_{QWP}|H\rangle$$

When the equations are subtituted into that form, It can be written as:

$$\begin{pmatrix}1 & 0\end{pmatrix}\frac{1}{\sqrt{2}}\begin{pmatrix}i-\cos 2\varphi & \sin 2\varphi \\ \sin 2\varphi & i+\cos 2\varphi\end{pmatrix}\begin{pmatrix}1 & 0 \\ 0 & 0\end{pmatrix}\frac{1}{\sqrt{2}}\begin{pmatrix}i-\cos 2\varphi & \sin 2\varphi \\ \sin 2\varphi & i+\cos 2\varphi\end{pmatrix}\begin{pmatrix}1 \\ 0\end{pmatrix}$$

And the fuction is:

$$\frac{1+\cos^2 2\varphi}{2}$$

This fuction represent the blue curve (H to QWP). The third graph is described by this form:

$$\langle V|\hat{O}^{\dagger}_{QWP}\hat{O}_{\rho H}\hat{O}_{QWP}|V\rangle$$

When the equations are subtituted into that form, It can be written as:

$$\begin{pmatrix}0 & 1\end{pmatrix}\frac{1}{\sqrt{2}}\begin{pmatrix}i-\cos 2\varphi & \sin 2\varphi \\ \sin 2\varphi & i+\cos 2\varphi\end{pmatrix}\begin{pmatrix}1 & 0 \\ 0 & 0\end{pmatrix}\frac{1}{\sqrt{2}}\begin{pmatrix}i-\cos 2\varphi & \sin 2\varphi \\ \sin 2\varphi & i+\cos 2\varphi\end{pmatrix}\begin{pmatrix}0 \\ 1\end{pmatrix}$$

And the fuction is:

$$\frac{1-\cos^2 2\varphi}{2}$$

This fuction represent the red curve (V to QWP). The fourth graph is described by this form:

$$\langle R|\hat{O}^{\dagger}_{QWP}\hat{O}_{\rho H}\hat{O}_{QWP}|R\rangle$$

When the equations are subtituted into that form, It can be written as:

$$\frac{1}{\sqrt{2}}\begin{pmatrix}1 & i\end{pmatrix}\frac{1}{\sqrt{2}}\begin{pmatrix}i-\cos 2\varphi & \sin 2\varphi \\ \sin 2\varphi & i+\cos 2\varphi\end{pmatrix}\begin{pmatrix}1 & 0 \\ 0 & 0\end{pmatrix}\frac{1}{\sqrt{2}}\begin{pmatrix}i-\cos 2\varphi & \sin 2\varphi \\ \sin 2\varphi & i+\cos 2\varphi\end{pmatrix}\frac{1}{\sqrt{2}}\begin{pmatrix}1 \\ -i\end{pmatrix}$$

And the fuction is:

$$\frac{1-\sin 2\varphi}{2}$$

This fuction represent the black curve (R to HWP).

Preparation

1. Optical devices

   In this work, there are four calculations to callibrate our theories and our experiments. Before doing the measurement, we need to callibrate our optical device. Some main optical devices are needed to measure state tomography are Beam Splitter (BS), Polarizing Beam Splitter (PBS), Photon Detector, Half Wave Plate (HWP), Quarter Wave Plate (QWP), Laser beam.

2. Callibration

   The optical devices will be arranged base on the measurement set-ups. And for this project, the measurement of number of photon should be proportional with our theory that we use. The measurements of photon's numbers will show the graph as follows:

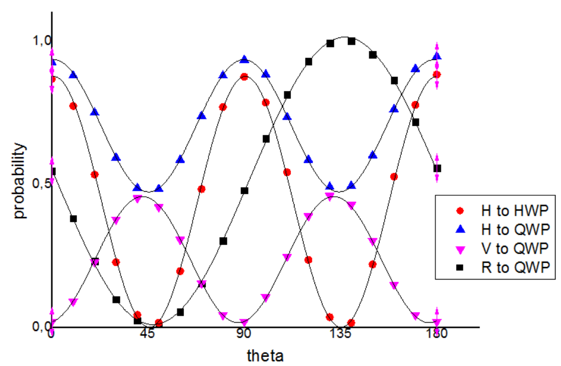

Figure 2 Relation between probabilty and angle of waveplate (base on callibration)

   This graph have same form with Figure 2 (The graph of relation between probability and angle of waveplate). This is showing four measurements that the polarized lights was passing trought the waveplate.

3. State tomography measurements

The number of photons with different set-ups will be measured to determine the density matrix. Due to Stokes parameters are used in this case, so we can do four measurements for each case. Those are: measuring photons using the beam splitter (unpolarized), horizontal measurement $|H\rangle$, antidiagonal measurement $|A\rangle$, and right-circular measurement $|R\rangle$. The set-ups as follow:

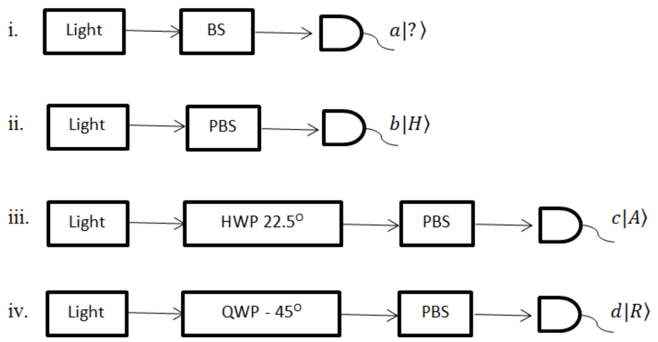

Figure 3. The tomography measurement set-up

From the measurement set-ups, n_0 will be found from the first set-up. n_1 will be found from the second set-up. n_2 will be found from the third set-up. and n_3 will be found from the fourth set-up.

Data analysis

After measuring n_i (n_0, n_1, n_2, n_3), The number of photons counted by the detectors are related to the Stokes parameters (James, On the Measurement of Qubits, 2008). And to calculate Stokes parameters, the equations are:

$$S_0 \equiv 2n_0$$
$$S_1 \equiv 2(n_1 - n_0)$$
$$S_2 \equiv 2(n_2 - n_0)$$
$$S_3 \equiv 2(n_3 - n_0)$$

With the Stokes parameters, the density matrix can be written as (Altepeter, James, & Kwiat, 2004):

$$\rho = \frac{1}{2S_0}\sum_{i=0}^{3} S_i \sigma_i$$

σ_i is Pauli matrices and the identity σ_0 as follows (Henao, 2017):

$$\sigma_0 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}; \sigma_1 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}; \sigma_2 = \begin{pmatrix} 0 & -1 \\ -1 & 1 \end{pmatrix}; \sigma_3 = \begin{pmatrix} 0 & i \\ -i & 0 \end{pmatrix}$$

If those matrices and Stokes parameters are substituted to the density matrix Equation, It can be written as:

$$\rho = \frac{1}{2S_0}\left[S_0 \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + S_1 \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} + S_2 \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix} + S_3 \begin{pmatrix} 0 & i \\ -i & 0 \end{pmatrix} \right]$$

And the simple form is:

$$\rho = \frac{1}{2S_0}\begin{pmatrix} S_0 + S_1 & -S_2 + iS_3 \\ -S_2 - iS_3 & S_0 - S_1 \end{pmatrix}$$

The density matrix is a result of this project which shows the mix state of the source in the experiment. The density matrix will be a picture which shown as diagram. And fidelity of the quantum state tomography is (Henao, 2017) :

$$F\left(\rho_{theory}, \rho_{experiment}\right) = \left(tr\sqrt{\sqrt{\rho_{theory}}\,\rho_{experiment}\sqrt{\rho_{theory}}} \right)^2$$

The fidelity will show us the closeness between ρ_{theory} and $\rho_{experiment}$. In this case, it shows the closseness between the experiment result and theory.

RESULTS AND DISCUSSION

Results

From the measurement, and It shows a numerical result shown by 2x2 matrix.

1. Density matrix of input $|H\rangle$

$$\rho_{H\ real} = \begin{pmatrix} 0.997 & -0.060 \\ -0.060 & 0.002 \end{pmatrix} \quad \rho_{H\ imaginary} = \begin{pmatrix} 0 & 0.029 \\ -0.029 & 0 \end{pmatrix}$$

2. Density matrix of input $|V\rangle$

$$\rho_{V\ real} = \begin{pmatrix} 0.004 & -0.073 \\ -0.073 & 0.995 \end{pmatrix} \quad \rho_{V\ imaginary} = \begin{pmatrix} 0 & 0.200 \\ -0.200 & 0 \end{pmatrix}$$

3. Density matrix of input $|D\rangle$

$$\rho_{D\ real} = \begin{pmatrix} 0.458 & 0.233 \\ 0.233 & 0.541 \end{pmatrix} \quad \rho_{D\ imaginary} = \begin{pmatrix} 0 & 0.033 \\ 0.033 & 0 \end{pmatrix}$$

4. Density matrix of input $|A\rangle$

$$\rho_{A\,\text{real}} = \begin{pmatrix} 0.583 & -0.241 \\ -0.241 & 0.416 \end{pmatrix} \quad \rho_{A\,\text{imaginary}} = \begin{pmatrix} 0 & -0.058 \\ 0.058 & 0 \end{pmatrix}$$

5. Density matrix of input $|R\rangle$

$$\rho_{R\,real} = \begin{pmatrix} 0.584 & -0.086 \\ -0.086 & 0.415 \end{pmatrix} \quad \rho_{R\,imaginary} = \begin{pmatrix} 0 & -0.060 \\ -0.060 & 0 \end{pmatrix}$$

6. Density matrix of input $|L\rangle$

$$\rho_{L\,\text{real}} = \begin{pmatrix} 0.592 & -0.137 \\ -0.137 & 0.407 \end{pmatrix} \quad \rho_{L\,imaginary} = \begin{pmatrix} 0 & -0.439 \\ 0.439 & 0 \end{pmatrix}$$

7. Laser IR 808nm

The last tomography in this research is laser beam without preparation state. It's mean that the source is unknown state. After measuring it, the density matrix is shown as follow:

$$\rho = \begin{pmatrix} 0.026 & 0.245 + 0.050i \\ 0.245 - 0.050i & 0.973 \end{pmatrix}$$

With the real part shown as bellow,

$$\rho_{real} = \begin{pmatrix} 0.026 & 0.245 \\ 0.245 & 0.973 \end{pmatrix}$$

And the imaginary part is,

$$\rho_{imaginary} = \begin{pmatrix} 0 & 0.050 \\ -0.050 & 0 \end{pmatrix}$$

All of them show the proportional value which approach the theory. The fidelity of the results shown by a table as follows,

Table 1: The fidelity of density matrices between the theory and experiment

Comparing density matrix	Fidelity
$\rho_{H\,experiment}$ **with** $\rho_{H\,theory}$	99.7 %
$\rho_{V\,experiment}$ **with** $\rho_{V\,theory}$	99.5 %
$\rho_{D\,experiment}$ **with** $\rho_{D\,theory}$	73.2 %
$\rho_{A\,experiment}$ **with** $\rho_{A\,theory}$	82.4 %
$\rho_{R\,experiment}$ **with** $\rho_{R\,theory}$	71.3 %
$\rho_{L\,experiment}$ **with** $\rho_{L\,theory}$	80.3 %

The fidelity is a measure of the closeness of two density matrices. In this case, the observer compared the density matrix from theory which have mentioned with the experiment results. These fidelities had verified that this work have a good agreement with the theory even though some samples is not show a very good result but all of them show the same form.

Discussion

The tomography sample that used in this work is the laser IR 808nm. Due to the outer product of a state has form 2x2 matrix, so the best way is choosing the samples which consist of horizontal state, vertical state, diagonal state, anti-diagonal state, right circular state, and left circular state. It means that the laser beam needs to be changed into the other polarization states by using the optical equipment that already callibrated. After getting the result for each samples, so the result can be compared with the theory. In other sample, the laser that used is unknown state, so it has been measured with a same method to know the state of that laser. This is a good think to know the unknown state of the laser which can be used to the other samples.

The results of this work show the 2x2 density matrix with maximum amplitude 1. So the value of density matrix will not be more than 1. In this work, the results consist of 14 images as tomography with a real part and imaginary part for each samples. The mixed state from the experiment shows 2x2 matrix which described as bellow:

$$\begin{array}{c c} \begin{array}{c} |H\rangle \\ |V\rangle \end{array} & \begin{pmatrix} A & B \\ C & D \end{pmatrix} \\ & \begin{array}{cc} |H\rangle & |V\rangle \end{array} \end{array}$$

A is a point for probability of horizontal state. So if the sample is horizontal input, it will show a high number in that point. B and C are the probabilities of the combination between $|H\rangle$ and $|V\rangle$. B and C can be the combinatino between a real and complex number. And D is the probablity for vertical state. It means that, if the input is changed to the vertical state as a sample, so the density matrix in point D would be highest number. When that matrix form is related with the result from the experiment, it will be showing part by part of the tomography from experiment.

The first result of this work shows the high number for the first sample (horizontal state tomography) which has already shown in figure 4.28. The input was polarized by using HWP with 0 degree to make horizontal polarization. The high number in the point A means that the first sample has a very good agreement with the theory. In other hand, for expected result, that point should have a value $\frac{1}{2}|H\rangle + \frac{1}{2}|H\rangle = A$. The high number of the point clarified that the tomography in the first sample was done well eventhough there are some noises in the other points. The second result also gives a good agreement. The sample is the laser beam was fired and put the HWP to polarize with rotating the angle to 45 degree. Based on the calculation, it shold be a vertical state. The result shows tha high value in the D point. The expected value of this case is $\frac{1}{2}|V\rangle + \frac{1}{2}|V\rangle = D$. Those results, the first and second sample, are differrent with diagonal state tomography and anti-diagonal state tomography. in those cases, the expected value is a half for all points in 2x2 matrix. The B and C were not a low value like the first and the second sample. But those points has $\frac{1}{\sqrt{2}}|H\rangle + \frac{1}{\sqrt{2}}|H\rangle = A$; $\frac{1}{\sqrt{2}}|H\rangle + \frac{1}{\sqrt{2}}|V\rangle = B$; $\frac{1}{\sqrt{2}}|H\rangle + \frac{1}{\sqrt{2}}|V\rangle = C$; and $\frac{1}{\sqrt{2}}|V\rangle + \frac{1}{\sqrt{2}}|V\rangle = D$. Diagonal and anti-diagonal state have the same value but doesn't have same representation. In this work, the density matrix of diagonal state was representated by positive calue in the B and C point but the anti-diagonal state was represented as negative value in the B and C point. The negative value is not a negative probability, but it is a repreasentation of the polarization direction. The results for diagonal and anti-diagonal have so low values in the points. This is happen due to the maximum and minimum wave amplitude is not consistent when the light was polarized with HWP any degree. So it might be there is any unpolarized light has been measured in the real experiment. for circular state tomography, the right and left circular was created by using QWP in order to change the direction from linear to circular form. The result of right circular state as input shows with value approach 0.5 for point A and D were represent the real part of the measurement. In the point b and c have form $a|H\rangle + b|V\rangle = C$ in the expected value with a and b can be complex number. In other hand, the left circular sample also shows the result with that form but it has negative value as $a|H\rangle + b|V\rangle = C$. The results from the experiment has a same form eventhough it is unperfect. For the last sample, laser beam which has unknown state had measured. It obtains the vertical state with fidelity 97.34%. In the point D with value 0.9734. It means that the state of laser beam has form approach $\frac{1}{2}|V\rangle + \frac{1}{2}|V\rangle = D$ in that part. In other hand, it has known if the laser will have maximum intensity if the beam propagates HWP with angle 45 degree and will have minimum intensity if the beam propagates HWP with angle 0 degree because by rotating the HWP, it will be changed the direction of wave which has effects to the output.

CONCLUTIONS

This project has shown the descriptions and results of single qubit quantum state tomography using density matrix with samples horizontal state, vertical state, diagonal state, anti-diagonal state, right circular state, left circular state, and directly from a laser IR 808nm which shows vertical signal. The fidelity of density matrix verifies that the experiment results have a good agreement with the theory. The experiment results are mixed state which can be represented by using a density matrix. In this work, the density matrice for all samples has found using optical devices and quantum mechanics approach which the results already shown that consist of real and imaginary part.

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